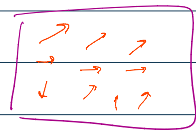
Math 53, Discussion 107

Monday, April 6, 2020 9:02 AM

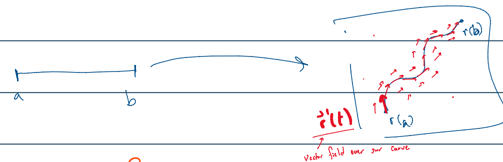
Chapter 16.1 (vector fields), Chapter 16.2 (line integrals)

A vector field on R^2 is: a function $\mathbb{R}^2 \to \mathbb{R}$ like wind on a plane

$\mathbf{F}(x,y)$ ← a vector



A parametrized curve is a function $\vec{r}(t): \mathbb{R}^1 \to \mathbb{R}^2$, $a \le t \le b$



A vector field on R^3 is: A function $\mathbb{R}^3 \to \mathbb{R}^3$

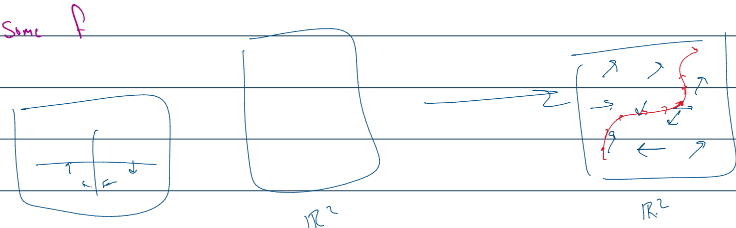
The gradient of a function is: a vector field

$\nabla f(x,y) : \mathbb{R}^2 \to \mathbb{R}^2$

A conservative vector field is: Some vector field $F(x,y)$ s.t. $F(x,y) = \nabla f(x,y)$ for some f

A vector field is a function $F: \mathbb{R}^2 \to \mathbb{R}^2$

in 1-D: $F(x) = \frac{d}{dx} f(x)$



To integrate a scalar function along a curve C, we write:

$\vec{r}(t) = (x(t), y(t))$ — can be written $\vec{r}(t): \mathbb{R}^1 \to \mathbb{R}^2$, $a \le t \le b$

$$\int_C f(x,y)ds = \int_a^b f(x(t),y(t))\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

(plug in param; speed of curve)

To integrate a vector field along a curve C we have:

$\vec{r}(t): \mathbb{R}^1 \to \mathbb{R}^3$, $a \le t \le b$; $F = (P(x,y,z), Q(x,y,z), R(x,y,z))$

$$\int_C F \cdot dr = \int_a^b F(r(t))\cdot r'(t)dt = \int_C F \cdot T\, ds = \int_C Pdx + Pdy + Pdz$$

Where $F = \langle P, Q, R \rangle$

$\frac{d}{dt}r = r'(t)$, $dr = r'(t)dt$; $T = \frac{r'(t)}{|r'(t)|}$

$\int_C Pdx = \int_a^b P(r(t))\,x'(t)\,dt$, $r(t) = (x,y,z)$

Ex $\vec{F}(x,y,z) = \sin(x)\hat{i} + xyz\hat{j} + \log(y)\hat{k}$

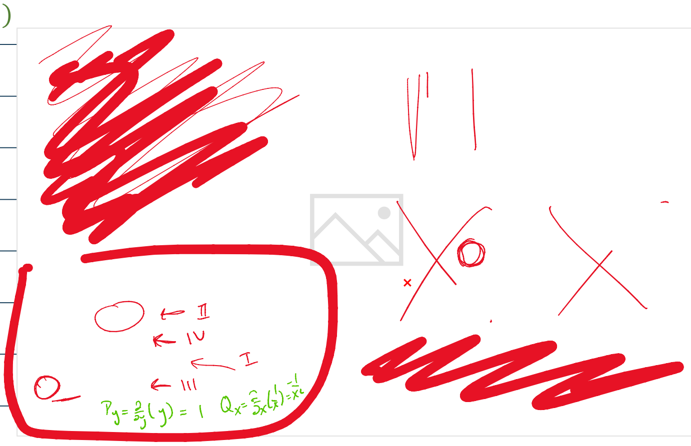
$P = \langle 1,0,0\rangle \cdot F = e_1 \cdot F$

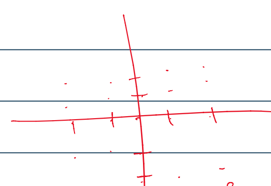
$Pdx + Qdy + Rdz = (P,Q,R)\cdot(dx,dy,dz) = F\cdot(dx,dy,dz) = F \cdot dr$

$\frac{dr}{dt} = \frac{1}{dt}\langle dx, dy, dz\rangle$, $r'(t) = (x', y', z')$

Exercises:

1)





$(P,Q) = F = \nabla f = (f_x, f_y)$; $P_y = f_{xy}$, $Q_x = f_{yx}$; $f_{xy} = f_{yx}$ (Clairaut) — should be same

If F is conservative $\Rightarrow P_y = Q_x$

2) $\int_C xy^4 ds$ with C the right half of the circle $x^2 + y^2 = 16$

3) $\int_C F \cdot dr$ with $F = \sin(x)\,i + \cos(y)\,j + xz\,k$ and $r = t^2 i - t^2 j + tk$, $0 \le t \le 1$

4) Compute $\int_C (1+xy)e^{xy}dx + (x^2e^{xy} + x)dy$ with C the unit circle. $= 0$

$t \leq 1$

4)Compute $\int_C (1 + xy)e^{xy}dx + (x^2 e^{xy} + x)dy$ with C the unit circle. $= 0$

(Also find a $f(x, y)$ such that ∇f is the integrand).

if $F = \nabla f$

$\int_C F \cdot dr = f(r(b)) - f(r(a))$

r is param of C

$a \leq t \leq b$

$\int_a^b \frac{d}{dx} f = f(b) - f(a)$

2) $\vec{r}(t) = \langle 4\cos(t), 4\sin(t) \rangle \quad -\frac{\pi}{2} \leq t \leq \pi/2$

$$\int_{-\pi/2}^{\pi/2} \underbrace{4\cos(t)}_{x} \underbrace{(4\sin(t))^4}_{y^4} \underbrace{\sqrt{4^2\sin^2(t) + 4^2\cos^2(t)}}_{4} \, dt$$

3) $\int_0^1 \langle \sin(t^2), \cos(-t^2), t^2 \cdot t \rangle \cdot \langle 2t, -2t, 1 \rangle \, dt$

$\int_0^1 2t\sin(t^2) - 2t\cos(-t^2) + t^3 \, dt$

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